Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 1 [Pure Mathematics 1]

May/June 2015 – February/March 2022

Chapter 8

Integration

441. 9709_m22_qp_12 Q: 1

A curve with equation $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 2x^{-\frac{1}{3}} - x^{\frac{1}{3}}$. It is given that $\mathrm{f}(8) = 5$.

Find $\mathrm{f}(x)$. [4]

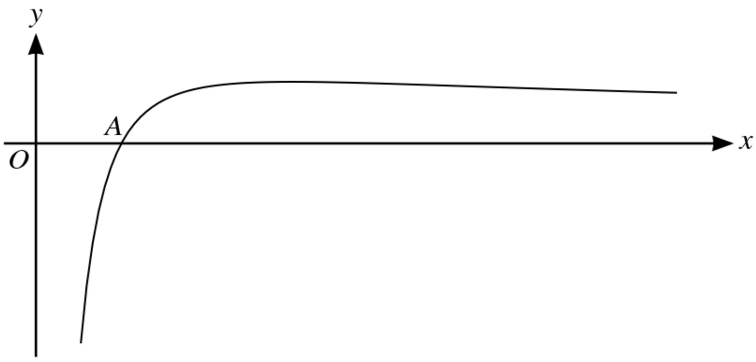
442. 9709_m21_qp_12 Q: 6

A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and $A\,(1, -3)$ lies on the curve. A point is moving along the curve and at A the y-coordinate of the point is increasing at 3 units per second.

(a) Find the rate of increase at A of the x-coordinate of the point. [3]

(b) Find the equation of the curve. [4]

443. 9709_m21_qp_12 Q: 11



The diagram shows the curve with equation $y = 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)$. The curve crosses the x-axis at the point A.

(a) Find the x-coordinate of A. [2]

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(b) Find the equation of the tangent to the curve at A. [4]

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(c) Find the x-coordinate of the maximum point of the curve. [2]

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(d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$. [4]

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If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

444. 9709_s21_qp_11 Q: 1

The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{x^4} + 32x^3$. It is given that the curve passes through the point $(\frac{1}{2}, 4)$.

Find the equation of the curve. [4]

445. 9709_s21_qp_11 Q: 11

The equation of a curve is $y = 2\sqrt{3x+4} - x$.

(a) Find the equation of the normal to the curve at the point (4, 4), giving your answer in the form $y = mx + c$. [5]

(b) Find the coordinates of the stationary point. [3]

(c) Determine the nature of the stationary point. [2]

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(d) Find the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$. [4]

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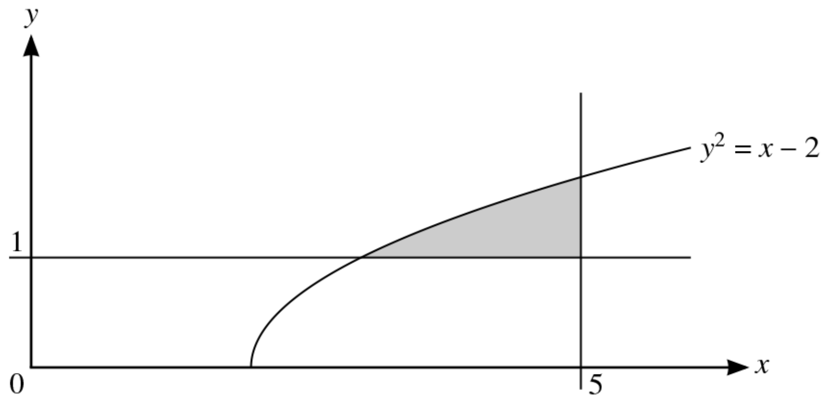
446. 9709_s21_qp_12 Q: 9



The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x-axis.

Find the volume obtained. [6]

447. 9709_s21_qp_12 Q: 11

The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

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(b) Find the equation of the curve. [4]

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(c) Find $\frac{d^2y}{dx^2}$. [2]

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(d) Determine the nature of the stationary point at (2, −3.5). [2]

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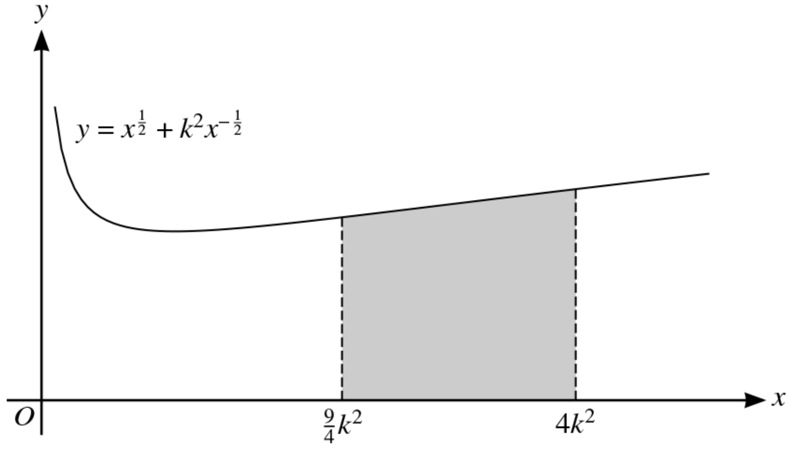
448. 9709_s21_qp_13 Q: 1

A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point $(2, 7)$.

Find $f(x)$. [3]

449. 9709_s21_qp_13 Q: 11



The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

(a) Find the coordinates of the minimum point of the curve, giving your answer in terms of k. [4]

The tangent at the point on the curve where $x = 4k^2$ intersects the y-axis at P.

(b) Find the y-coordinate of P in terms of k. [4]

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The shaded region is bounded by the curve, the x-axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

(c) Find the area of the shaded region in terms of k. [3]

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If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

450. 9709_w21_qp_11 Q: 9

A curve has equation $y = f(x)$, and it is given that $f'(x) = 2x^2 - 7 - \frac{4}{x^2}$.

(a) Given that $f(1) = -\frac{1}{3}$, find $f(x)$. [4]

(b) Find the coordinates of the stationary points on the curve. [5]

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(c) Find $f''(x)$. [1]

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(d) Hence, or otherwise, determine the nature of each of the stationary points. [2]

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451. 9709_w21_qp_11 Q: 10

(a) Find $\int_{1}^{\infty} \frac{1}{(3x-2)^{\frac{3}{2}}}\, dx$. [4]

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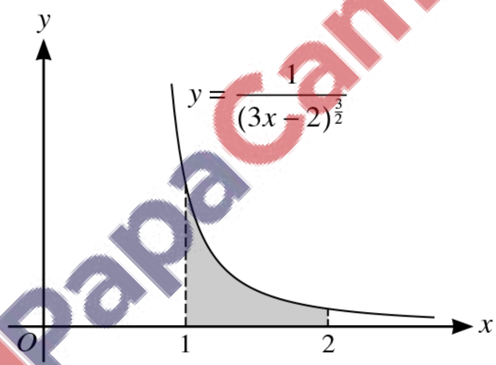

The diagram shows the curve with equation $y = \frac{1}{(3x-2)^{\frac{3}{2}}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. The shaded region is rotated through 360° about the x-axis.

(b) Find the volume of revolution. [4]

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The normal to the curve at the point $(1,\ 1)$ crosses the y-axis at the point A.

(c) Find the y-coordinate of A. [4]

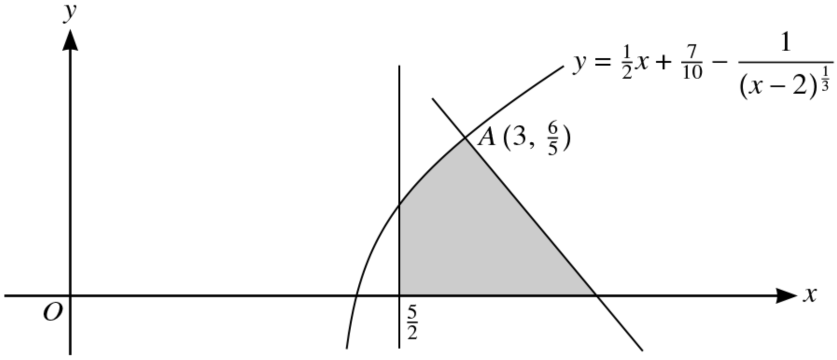
If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

452. 9709_w21_qp_12 Q: 4

A curve is such that $\frac{dy}{dx} = \frac{8}{(3x+2)^2}$. The curve passes through the point $(2, 5\frac{2}{3})$.

Find the equation of the curve. [4]

453. 9709_w21_qp_12 Q: 11

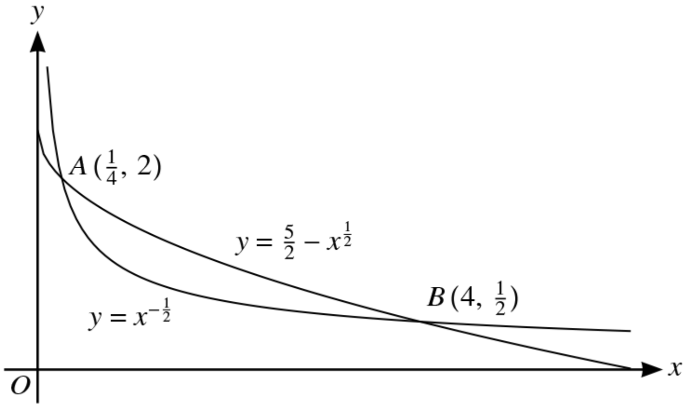


The diagram shows the line $x = \frac{5}{2}$, part of the curve $y = \frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}}$ and the normal to the curve at the point $A\ \left(3,\ \frac{6}{5}\right)$.

(a) Find the x-coordinate of the point where the normal to the curve meets the x-axis. [5]

(b) Find the area of the shaded region, giving your answer correct to 2 decimal places. [6]

454. 9709_w21_qp_13 Q: 8



The diagram shows the curves with equations $y = x^{-\frac{1}{2}}$ and $y = \frac{5}{2} - x^{\frac{1}{2}}$. The curves intersect at the points $A\left(\frac{1}{4}, 2\right)$ and $B\left(4, \frac{1}{2}\right)$.

(a) Find the area of the region between the two curves. [6]

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(b) The normal to the curve $y = x^{-\frac{1}{2}}$ at the point (1, 1) intersects the y-axis at the point (0, p).

Find the value of p. [4]

455. 9709_w21_qp_13 Q: 10

A curve has equation $y = f(x)$ and it is given that

$$f'(x) = \left(\tfrac{1}{2}x + k\right)^{-2} - (1 + k)^{-2},$$

where k is a constant. The curve has a minimum point at $x = 2$.

(a) Find $f''(x)$ in terms of k and x, and hence find the set of possible values of k. [3]

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It is now given that $k = -3$ and the minimum point is at $(2, 3\frac{1}{2})$.

(b) Find $f(x)$. [4]

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(c) Find the coordinates of the other stationary point and determine its nature. [4]

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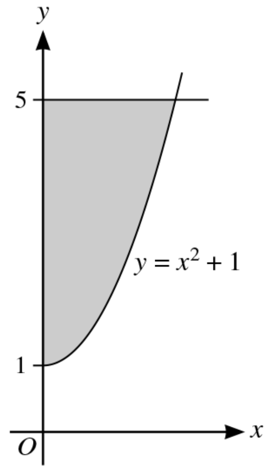
456. 9709_m20_qp_12 Q: 3



The diagram shows part of the curve with equation $y = x^2 + 1$. The shaded region enclosed by the curve, the y-axis and the line $y = 5$ is rotated through 360° about the ***y*-axis**.

Find the volume obtained. [4]

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457. 9709_m20_qp_12 Q: 10

The gradient of a curve at the point (x, y) is given by $\frac{dy}{dx} = 2(x+3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where a is a positive constant.

(a) Find the value of a. [3]

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(b) Determine the nature of the stationary point. [3]

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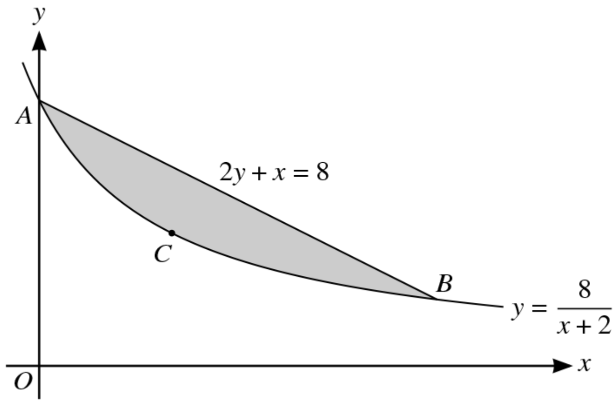
(c) Find the equation of the curve. [4]

458. 9709_s20_qp_11 Q: 11

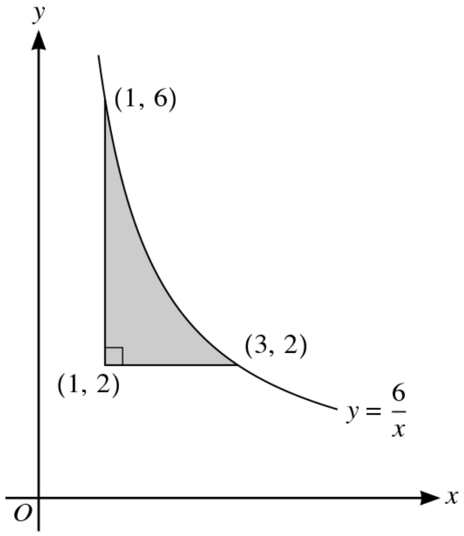


The diagram shows part of the curve $y = \frac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

(a) Find, by calculation, the coordinates of A, B and C. [6]

(b) Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through 360° about the x-axis. [6]

459. 9709_s20_qp_12 Q: 8



The diagram shows part of the curve $y = \frac{6}{x}$. The points (1, 6) and (3, 2) lie on the curve. The shaded region is bounded by the curve and the lines $y = 2$ and $x = 1$.

(a) Find the volume generated when the shaded region is rotated through 360° about the ***y*-axis**. [5]

(b) The tangent to the curve at a point X is parallel to the line $y + 2x = 0$. Show that X lies on the line $y = 2x$. [3]

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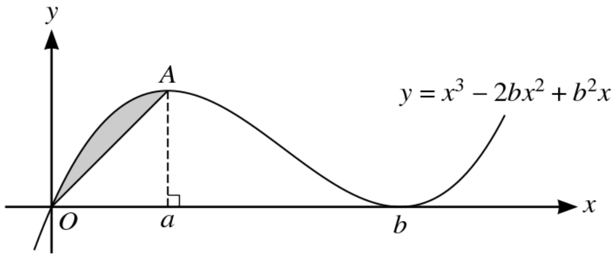
460. 9709_s20_qp_13 Q: 2

The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. It is given that the point (4, 7) lies on the curve.

Find the equation of the curve. [4]

461. 9709_s20_qp_13 Q: 11



The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA, where A is the maximum point on the curve. The x-coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

(a) Show that $b = 3a$. [4]

(b) Show that the area of the shaded region between the line and the curve is ka^4, where k is a fraction to be found. [7]

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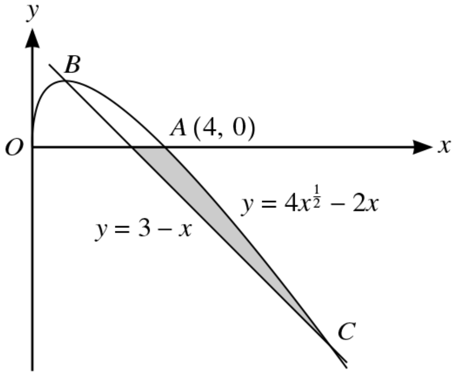
462. 9709_w20_qp_11 Q: 2

The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{(x-3)^2} + x$. It is given that the curve passes through the point (2, 7).

Find the equation of the curve. [4]

463. 9709_w20_qp_11 Q: 12



The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geqslant 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A\,(4, 0)$ and crosses the straight line at B and C.

(a) Find, by calculation, the x-coordinates of B and C. [4]

(b) Show that B is a stationary point on the curve. [2]

(c) Find the area of the shaded region. [6]

464. 9709_w20_qp_12 Q: 7

The point (4, 7) lies on the curve $y = f(x)$ and it is given that $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(a) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the y-coordinate when $x = 4$. [3]

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(b) Find the equation of the curve. [4]

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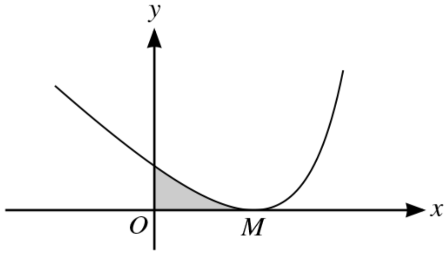
465. 9709_w20_qp_12 Q: 10



The diagram shows part of the curve $y = \frac{2}{(3-2x)^2} - x$ and its minimum point M, which lies on the x-axis.

(a) Find expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\int y\,dx$. [6]

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(b) Find, by calculation, the x-coordinate of M. [2]

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(c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

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466. 9709_w20_qp_13 Q: 2

The function f is defined by $f(x) = \dfrac{2}{(x+2)^2}$ for $x > -2$.

(a) Find $\displaystyle\int_1^{\infty} f(x)\,dx$. [3]

(b) The equation of a curve is such that $\dfrac{dy}{dx} = f(x)$. It is given that the point $(-1, -1)$ lies on the curve.

Find the equation of the curve. [2]

467. 9709_w20_qp_13 Q: 10

A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k. [4]

(b) It is given instead that $\int_{\frac{1}{4}k^2}^{k^2} \left(\frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2} \right) dx = \frac{13}{12}$.

Find the value of k. [5]

468. 9709_m19_qp_12 Q: 2

A curve with equation $y = \mathrm{f}(x)$ passes through the points (0, 2) and (3, −1). It is given that $\mathrm{f}'(x) = kx^2 - 2x$, where k is a constant. Find the value of k. [5]

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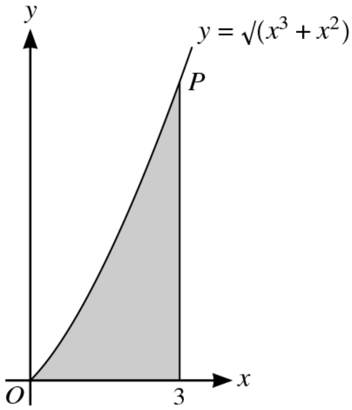
469. 9709_m19_qp_12 Q: 9



The diagram shows part of the curve with equation $y = \sqrt{(x^3 + x^2)}$. The shaded region is bounded by the curve, the x-axis and the line $x = 3$.

(i) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

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(ii) P is the point on the curve with x-coordinate 3. Find the y-coordinate of the point where the normal to the curve at P crosses the y-axis. [6]

470. 9709_s19_qp_11 Q: 10

A curve for which $\frac{d^2y}{dx^2} = 2x - 5$ has a stationary point at (3, 6).

(i) Find the equation of the curve. [6]

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(ii) Find the x-coordinate of the other stationary point on the curve. [1]

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(iii) Determine the nature of each of the stationary points. [2]

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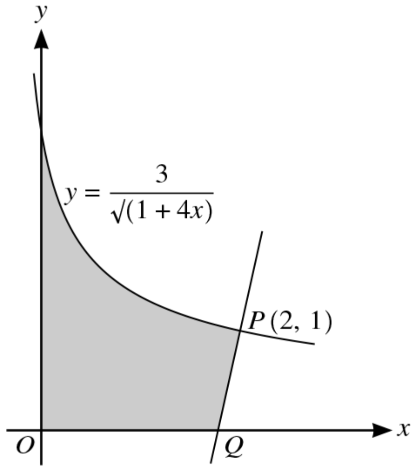
471. 9709_s19_qp_11 Q: 11



The diagram shows part of the curve $y = \frac{3}{\sqrt{(1+4x)}}$ and a point $P(2, 1)$ lying on the curve. The normal to the curve at P intersects the x-axis at Q.

(i) Show that the x-coordinate of Q is $\frac{16}{9}$. [5]

(ii) Find, showing all necessary working, the area of the shaded region. [6]

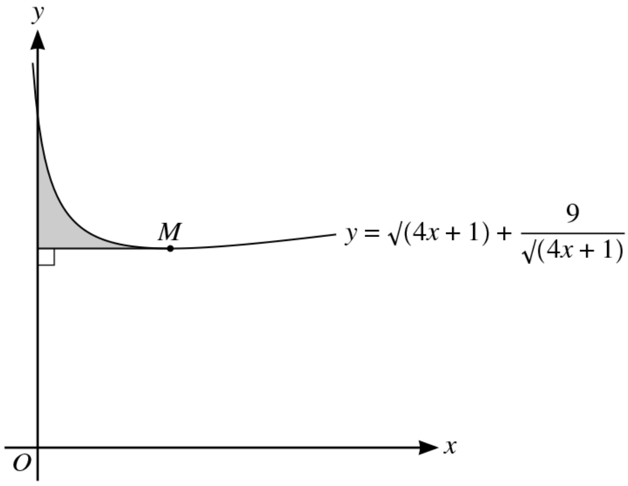
472. 9709_s19_qp_12 Q: 3

A curve is such that $\frac{dy}{dx} = x^3 - \frac{4}{x^2}$. The point $P(2, 9)$ lies on the curve.

(i) A point moves on the curve in such a way that the x-coordinate is decreasing at a constant rate of 0.05 units per second. Find the rate of change of the y-coordinate when the point is at P. [2]

(ii) Find the equation of the curve. [3]

473. 9709_s19_qp_12 Q: 11



The diagram shows part of the curve $y = \sqrt{(4x+1)} + \dfrac{9}{\sqrt{(4x+1)}}$ and the minimum point M.

(i) Find expressions for $\dfrac{dy}{dx}$ and $\int y\,dx$. [6]

(ii) Find the coordinates of M. [3]

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The shaded region is bounded by the curve, the y-axis and the line through M parallel to the x-axis.

(iii) Find, showing all necessary working, the area of the shaded region. [3]

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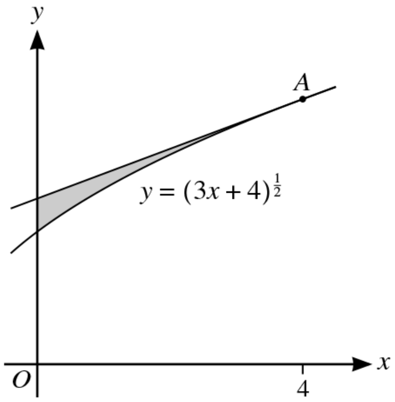
474. 9709_s19_qp_13 Q: 10



The diagram shows part of the curve with equation $y = (3x + 4)^{\frac{1}{2}}$ and the tangent to the curve at the point A. The x-coordinate of A is 4.

(i) Find the equation of the tangent to the curve at A. [5]

(ii) Find, showing all necessary working, the area of the shaded region. [5]

(iii) A point is moving along the curve. At the point P the y-coordinate is increasing at half the rate at which the x-coordinate is increasing. Find the x-coordinate of P. [3]

475. 9709_w19_qp_11 Q: 9

A curve for which $\frac{dy}{dx} = (5x - 1)^{\frac{1}{2}} - 2$ passes through the point (2, 3).

(i) Find the equation of the curve. [4]

(ii) Find $\frac{d^2y}{dx^2}$. [2]

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(iii) Find the coordinates of the stationary point on the curve and, showing all necessary working, determine the nature of this stationary point. [4]

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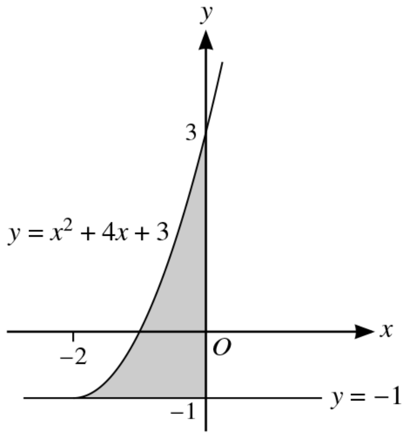
476. 9709_w19_qp_11 Q: 11



The diagram shows a shaded region bounded by the y-axis, the line $y = -1$ and the part of the curve $y = x^2 + 4x + 3$ for which $x \geqslant -2$.

(i) Express $y = x^2 + 4x + 3$ in the form $y = (x + a)^2 + b$, where a and b are constants. Hence, for $x \geqslant -2$, express x in terms of y. [4]

(ii) Hence, showing all necessary working, find the volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [6]

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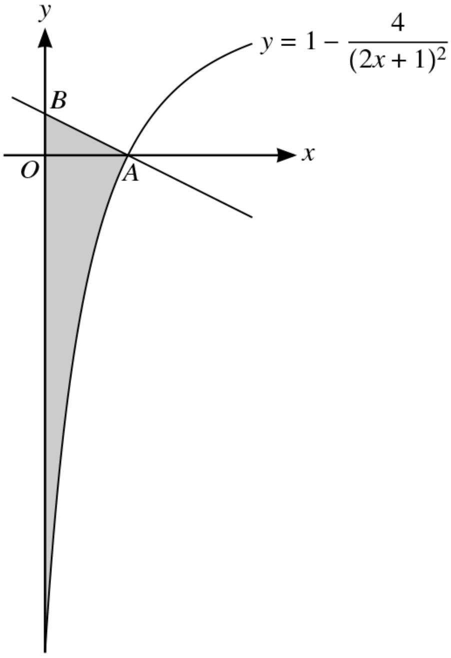
477. 9709_w19_qp_12 Q: 3

A curve is such that $\frac{dy}{dx} = \frac{k}{\sqrt{x}}$, where k is a constant. The points $P(1, -1)$ and $Q(4, 4)$ lie on the curve. Find the equation of the curve. [4]

478. 9709_w19_qp_12 Q: 10



The diagram shows part of the curve $y = 1 - \dfrac{4}{(2x+1)^2}$. The curve intersects the x-axis at A. The normal to the curve at A intersects the y-axis at B.

(i) Obtain expressions for $\dfrac{dy}{dx}$ and $\displaystyle\int y\,dx$. [4]

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(ii) Find the coordinates of B. [4]

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(iii) Find, showing all necessary working, the area of the shaded region. [4]

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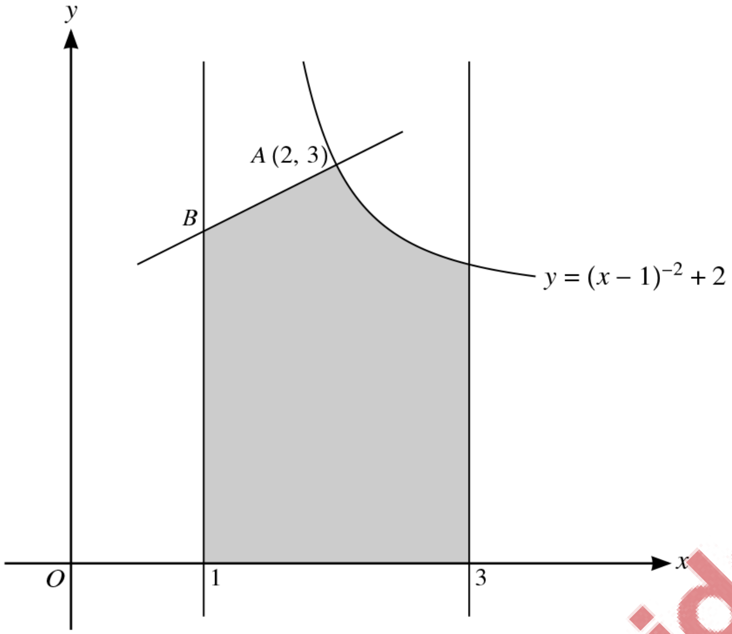
479. 9709_w19_qp_13 Q: 8

A function f is defined for $x > \frac{1}{2}$ and is such that $f'(x) = 3(2x - 1)^{\frac{1}{2}} - 6$.

(i) Find the set of values of x for which f is decreasing. [4]

(ii) It is now given that $f(1) = -3$. Find $f(x)$. [4]

480. 9709_w19_qp_13 Q: 11



The diagram shows part of the curve $y = (x-1)^{-2} + 2$, and the lines $x = 1$ and $x = 3$. The point A on the curve has coordinates $(2, 3)$. The normal to the curve at A crosses the line $x = 1$ at B.

(i) Show that the normal AB has equation $y = \frac{1}{2}x + 2$. [3]

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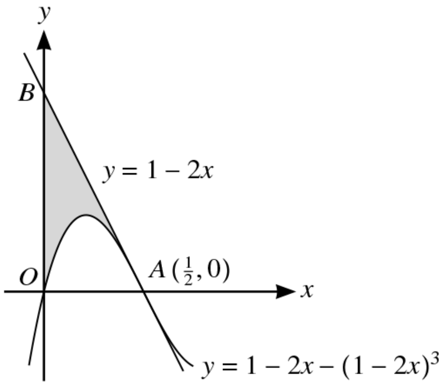
(ii) Find, showing all necessary working, the volume of revolution obtained when the shaded region is rotated through 360° about the x-axis. [8]

481. 9709_m18_qp_12 Q: 1

A curve passes through the point (4, −6) and has an equation for which $\frac{dy}{dx} = x^{-\frac{1}{2}} - 3$. Find the equation of the curve. [4]

482. 9709_m18_qp_12 Q: 11



The diagram shows part of the curve $y = 1 - 2x - (1 - 2x)^3$ intersecting the x-axis at the origin O and at $A\left(\frac{1}{2}, 0\right)$. The line AB intersects the y-axis at B and has equation $y = 1 - 2x$.

(i) Show that AB is the tangent to the curve at A. [4]

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(ii) Show that the area of the shaded region can be expressed as $\int_0^{\frac{1}{2}} (1-2x)^3 \, dx$. [2]

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(iii) Hence, showing all necessary working, find the area of the shaded region. [3]

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483. 9709_s18_qp_11 Q: 3

A curve is such that $\frac{dy}{dx} = \frac{12}{(2x+1)^2}$. The point (1, 1) lies on the curve. Find the coordinates of the point at which the curve intersects the x-axis. [6]

484. 9709_s18_qp_11 Q: 10

The curve with equation $y = x^3 - 2x^2 + 5x$ passes through the origin.

(i) Show that the curve has no stationary points. [3]

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(ii) Denoting the gradient of the curve by m, find the stationary value of m and determine its nature. [5]

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(iii) Showing all necessary working, find the area of the region enclosed by the curve, the x-axis and the line $x = 6$. [4]

485. 9709_s18_qp_12 Q: 9

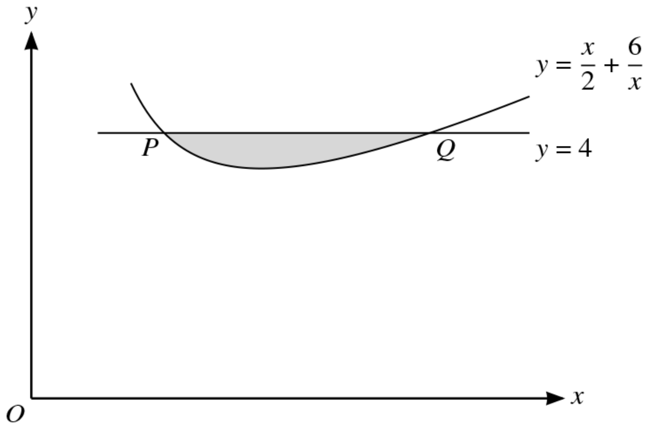
A curve is such that $\frac{dy}{dx} = \sqrt{(4x+1)}$ and $(2, 5)$ is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) A point P moves along the curve in such a way that the y-coordinate is increasing at a constant rate of 0.06 units per second. Find the rate of change of the x-coordinate when P passes through (2, 5). [2]

(iii) Show that $\frac{d^2y}{dx^2} \times \frac{dy}{dx}$ is constant. [2]

486. 9709_s18_qp_12 Q: 11



The diagram shows part of the curve $y = \frac{x}{2} + \frac{6}{x}$. The line $y = 4$ intersects the curve at the points P and Q.

(i) Show that the tangents to the curve at P and Q meet at a point on the line $y = x$. [6]

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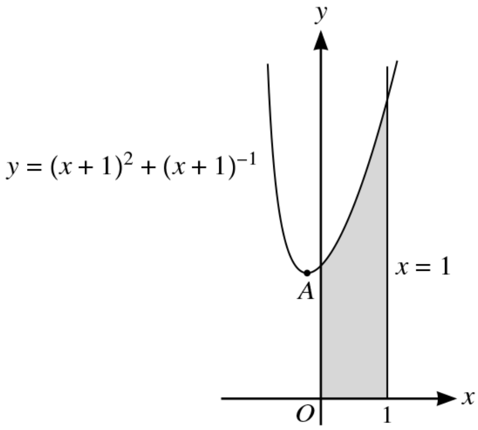
(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. Give your answer in terms of π. [6]

487. 9709_s18_qp_13 Q: 4

A curve with equation $y = \mathrm{f}(x)$ passes through the point $A\,(3,\ 1)$ and crosses the y-axis at B. It is given that $\mathrm{f}'(x) = (3x-1)^{-\frac{1}{3}}$. Find the y-coordinate of B. [6]

488. 9709_s18_qp_13 Q: 11



The diagram shows part of the curve $y = (x+1)^2 + (x+1)^{-1}$ and the line $x = 1$. The point A is the minimum point on the curve.

(i) Show that the x-coordinate of A satisfies the equation $2(x+1)^3 = 1$ and find the exact value of $\frac{d^2y}{dx^2}$ at A. [5]

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(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

489. 9709_w18_qp_11 Q: 6

A curve has a stationary point at $(3, 9\frac{1}{2})$ and has an equation for which $\frac{dy}{dx} = ax^2 + a^2x$, where a is a non-zero constant.

(i) Find the value of a. [2]

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(ii) Find the equation of the curve. [4]

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(iii) Determine, showing all necessary working, the nature of the stationary point. [2]

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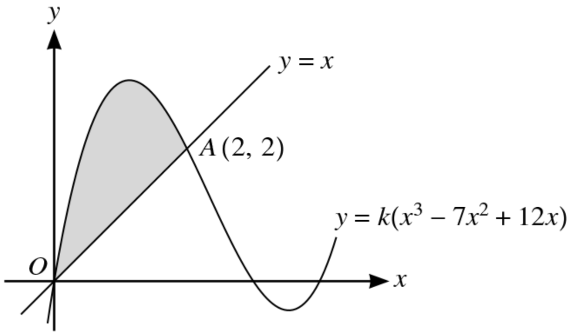
490. 9709_w18_qp_11 Q: 7



The diagram shows part of the curve with equation $y = k(x^3 - 7x^2 + 12x)$ for some constant k. The curve intersects the line $y = x$ at the origin O and at the point $A\,(2, 2)$.

(i) Find the value of k. [1]

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(ii) Verify that the curve meets the line $y = x$ again when $x = 5$. [2]

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(iii) Find, showing all necessary working, the area of the shaded region. [5]

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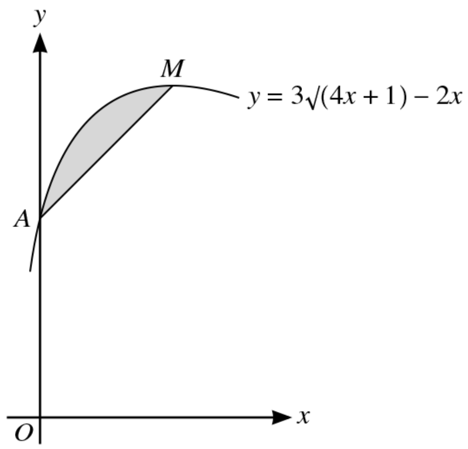
491. 9709_w18_qp_12 Q: 2

Showing all necessary working, find $\int_1^4 \left(\sqrt{x} + \frac{2}{\sqrt{x}}\right) dx$. [4]

492. 9709_w18_qp_12 Q: 11



The diagram shows part of the curve $y = 3\sqrt{(4x+1)} - 2x$. The curve crosses the y-axis at A and the stationary point on the curve is M.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\int y \, dx$. [5]

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(ii) Find the coordinates of M. [3]

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(iii) Find, showing all necessary working, the area of the shaded region. [4]

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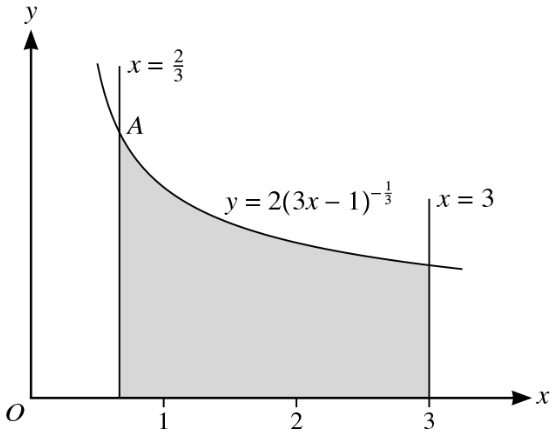
493. 9709_w18_qp_13 Q: 8

A curve passes through (0, 11) and has an equation for which $\frac{dy}{dx} = ax^2 + bx - 4$, where a and b are constants.

(i) Find the equation of the curve in terms of a and b. [3]

(ii) It is now given that the curve has a stationary point at (2, 3). Find the values of a and b. [5]

494. 9709_w18_qp_13 Q: 10

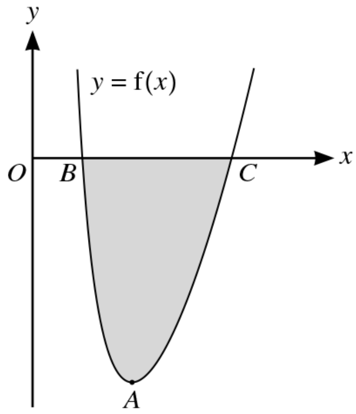


The diagram shows part of the curve $y = 2(3x - 1)^{-\frac{1}{3}}$ and the lines $x = \frac{2}{3}$ and $x = 3$. The curve and the line $x = \frac{2}{3}$ intersect at the point A.

(i) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [5]

(ii) Find the equation of the normal to the curve at A, giving your answer in the form $y = mx + c$. [5]

495. 9709_m17_qp_12 Q: 10



The diagram shows the curve $y = f(x)$ defined for $x > 0$. The curve has a minimum point at A and crosses the x-axis at B and C. It is given that $\frac{dy}{dx} = 2x - \frac{2}{x^3}$ and that the curve passes through the point $(4, \frac{189}{16})$.

(i) Find the x-coordinate of A. [2]

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(ii) Find $f(x)$. [3]

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(iii) Find the x-coordinates of B and C. [4]

(iv) Find, showing all necessary working, the area of the shaded region. [4]

496. 9709_s17_qp_11 Q: 7

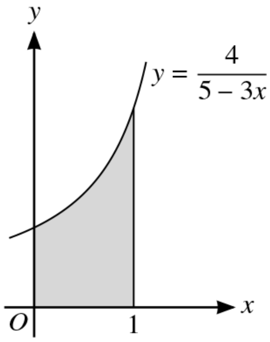
A curve for which $\frac{dy}{dx} = 7 - x^2 - 6x$ passes through the point $(3, -10)$.

(i) Find the equation of the curve. [3]

(ii) Express $7 - x^2 - 6x$ in the form $a - (x + b)^2$, where a and b are constants. [2]

(iii) Find the set of values of x for which the gradient of the curve is positive. [3]

497. 9709_s17_qp_11 Q: 10



The diagram shows part of the curve $y = \dfrac{4}{5-3x}$.

(i) Find the equation of the normal to the curve at the point where $x = 1$ in the form $y = mx + c$, where m and c are constants. [5]

The shaded region is bounded by the curve, the coordinate axes and the line $x = 1$.

(ii) Find, showing all necessary working, the volume obtained when this shaded region is rotated through $360°$ about the x-axis. [5]

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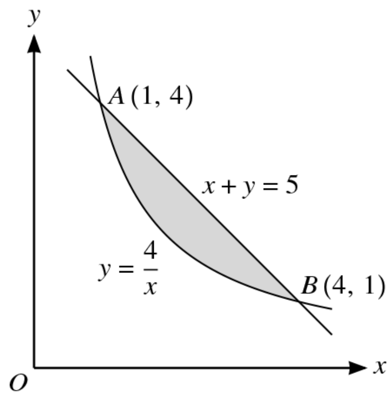
498. 9709_s17_qp_12 Q: 6



The diagram shows the straight line $x + y = 5$ intersecting the curve $y = \frac{4}{x}$ at the points $A\,(1, 4)$ and $B\,(4, 1)$. Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [7]

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499. 9709_s17_qp_13 Q: 10

(a)

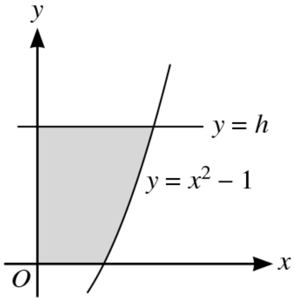

Fig. 1

Fig. 1 shows part of the curve $y = x^2 - 1$ and the line $y = h$, where h is a constant.

(i) The shaded region is rotated through 360° about the ***y*-axis**. Show that the volume of revolution, V, is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. [3]

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(ii) Find, showing all necessary working, the area of the shaded region when $h = 3$. [4]

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(b)

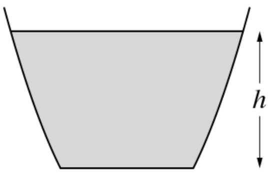

Fig. 2

Fig. 2 shows a cross-section of a bowl containing water. When the height of the water level is h cm, the volume, V cm^3, of water is given by $V = \pi(\frac{1}{2}h^2 + h)$. Water is poured into the bowl at a constant rate of 2 cm^3 s^{-1}. Find the rate, in cm s^{-1}, at which the height of the water level is increasing when the height of the water level is 3 cm. [4]

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500. 9709_s17_qp_13 Q: 11

The function f is defined for $x \geqslant 0$. It is given that f has a minimum value when $x = 2$ and that $f''(x) = (4x + 1)^{-\frac{1}{2}}$.

(i) Find $f'(x)$. [3]

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It is now given that $f''(0)$, $f'(0)$ and $f(0)$ are the first three terms respectively of an arithmetic progression.

(ii) Find the value of $f(0)$. [3]

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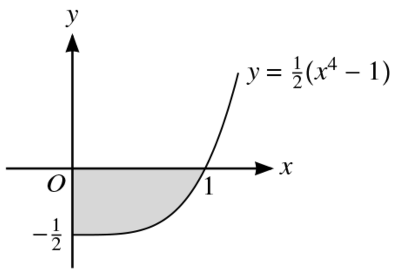
(iii) Find $f(x)$, and hence find the minimum value of f. [5]

501. 9709_w17_qp_11 Q: 10



The diagram shows part of the curve $y = \frac{1}{2}(x^4 - 1)$, defined for $x \geqslant 0$.

(i) Find, showing all necessary working, the area of the shaded region. [3]

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(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

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(iii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the y-axis. [5]

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502. 9709_w17_qp_12 Q: 8

A curve is such that $\frac{dy}{dx} = -x^2 + 5x - 4$.

(i) Find the x-coordinate of each of the stationary points of the curve. [2]

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(ii) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence or otherwise find the nature of each of the stationary points. [3]

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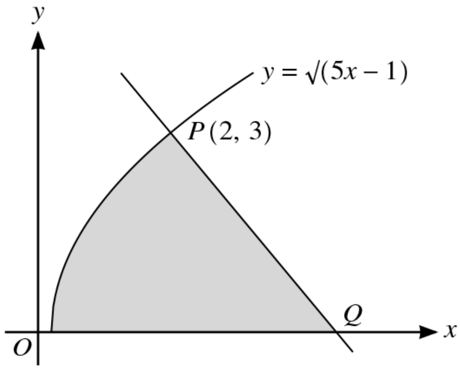
(iii) Given that the curve passes through the point (6, 2), find the equation of the curve. [4]

503. 9709_w17_qp_12 Q: 10



The diagram shows part of the curve $y = \sqrt{(5x - 1)}$ and the normal to the curve at the point $P(2, 3)$. This normal meets the x-axis at Q.

(i) Find the equation of the normal at P. [4]

(ii) Find, showing all necessary working, the area of the shaded region. [7]

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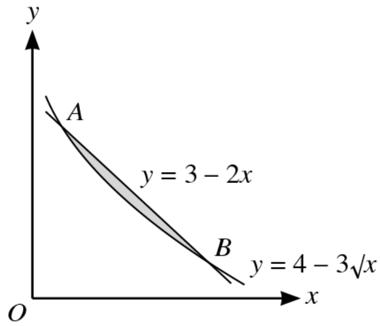
504. 9709_w17_qp_13 Q: 8



The diagram shows parts of the graphs of $y = 3 - 2x$ and $y = 4 - 3\sqrt{x}$ intersecting at points A and B.

(i) Find by calculation the x-coordinates of A and B. [3]

(ii) Find, showing all necessary working, the area of the shaded region. [5]

505. 9709_w17_qp_13 Q: 10

A curve has equation $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = ax^2 + bx$, where a and b are positive constants.

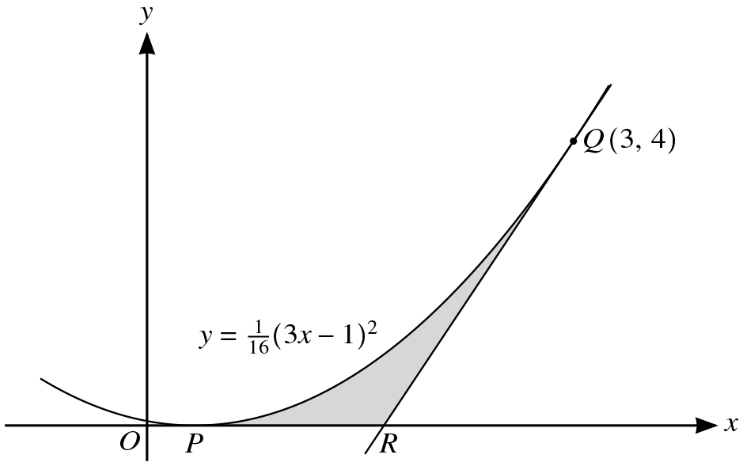
(i) Find, in terms of a and b, the non-zero value of x for which the curve has a stationary point and determine, showing all necessary working, the nature of the stationary point. [3]

(ii) It is now given that the curve has a stationary point at $(-2, -3)$ and that the gradient of the curve at $x = 1$ is 9. Find $f(x)$. [6]

506. 9709_m16_qp_12 Q: 2

A curve for which $\frac{dy}{dx} = 3x^2 - \frac{2}{x^3}$ passes through $(-1, 3)$. Find the equation of the curve. [4]

507. 9709_m16_qp_12 Q: 10



The diagram shows part of the curve $y = \frac{1}{16}(3x - 1)^2$, which touches the x-axis at the point P. The point $Q\,(3, 4)$ lies on the curve and the tangent to the curve at Q crosses the x-axis at R.

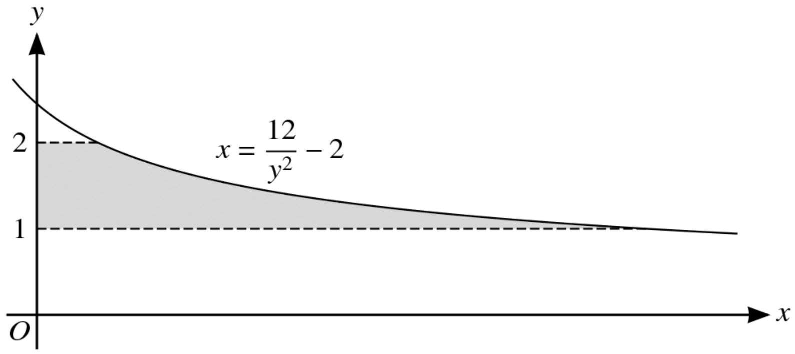
(i) State the x-coordinate of P. [1]

Showing all necessary working, find by calculation

(ii) the x-coordinate of R, [5]

(iii) the area of the shaded region PQR. [6]

508. 9709_s16_qp_11 Q: 3



The diagram shows part of the curve $x = \frac{12}{y^2} - 2$. The shaded region is bounded by the curve, the y-axis and the lines $y = 1$ and $y = 2$. Showing all necessary working, find the volume, in terms of π, when this shaded region is rotated through 360° about the y-axis. [5]

509. 9709_s16_qp_11 Q: 4

A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2 - 8(3x+4)^{-\frac{1}{2}}$.

(i) A point P moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.3 units per second. Find the rate of change of the y-coordinate as P crosses the y-axis. [2]

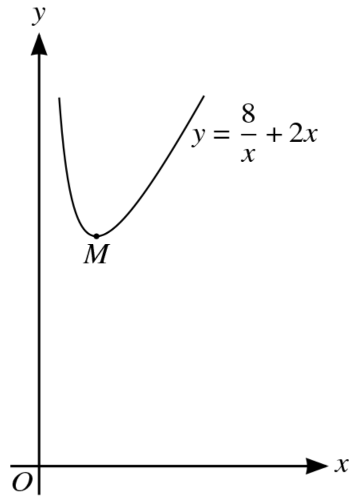
The curve intersects the y-axis where $y = \frac{4}{3}$.

(ii) Find the equation of the curve. [4]

510. 9709_s16_qp_12 Q: 2

A curve is such that $\frac{dy}{dx} = \frac{8}{(5-2x)^2}$. Given that the curve passes through (2, 7), find the equation of the curve. [4]

511. 9709_s16_qp_12 Q: 10



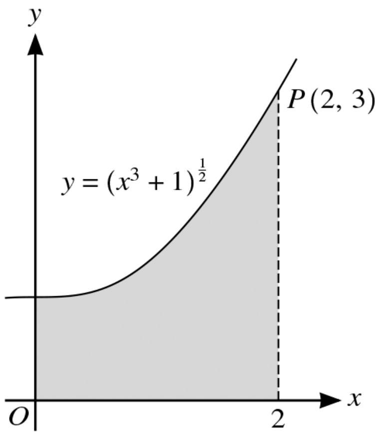
The diagram shows the part of the curve $y = \frac{8}{x} + 2x$ for $x > 0$, and the minimum point M.

(i) Find expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\int y^2\,dx$. [5]

(ii) Find the coordinates of M and determine the coordinates and nature of the stationary point on the part of the curve for which $x < 0$. [5]

(iii) Find the volume obtained when the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. [2]

512. 9709_s16_qp_13 Q: 2



The diagram shows part of the curve $y = (x^3 + 1)^{\frac{1}{2}}$ and the point $P\,(2, 3)$ lying on the curve. Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

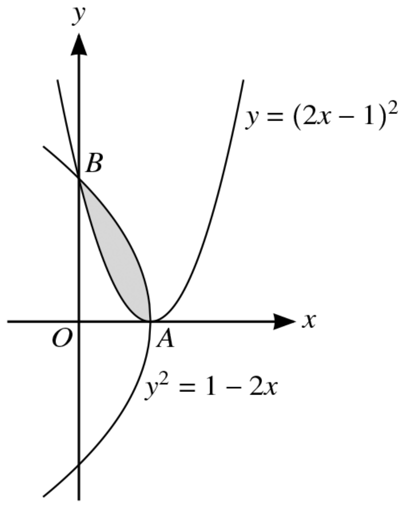
513. 9709_s16_qp_13 Q: 3

A curve is such that $\frac{dy}{dx} = 6x^2 + \frac{k}{x^3}$ and passes through the point $P(1, 9)$. The gradient of the curve at P is 2.

(i) Find the value of the constant k. [1]

(ii) Find the equation of the curve. [4]

514. 9709_w16_qp_11 Q: 7



The diagram shows parts of the curves $y = (2x - 1)^2$ and $y^2 = 1 - 2x$, intersecting at points A and B.

(i) State the coordinates of A. [1]

(ii) Find, showing all necessary working, the area of the shaded region. [6]

515. 9709_w16_qp_11 Q: 10

A curve has equation $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = 3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$. The point A is the only point on the curve at which the gradient is -1.

(i) Find the x-coordinate of A. [3]

(ii) Given that the curve also passes through the point $(4, 10)$, find the y-coordinate of A, giving your answer as a fraction. [6]

516. 9709_w16_qp_12 Q: 1

A curve is such that $\frac{dy}{dx} = \frac{8}{\sqrt{(4x+1)}}$. The point (2, 5) lies on the curve. Find the equation of the curve. [4]

517. 9709_w16_qp_13 Q: 10

A curve is such that $\frac{dy}{dx} = \frac{2}{a}x^{-\frac{1}{2}} + ax^{-\frac{3}{2}}$, where a is a positive constant. The point $A\,(a^2, 3)$ lies on the curve. Find, in terms of a,

(i) the equation of the tangent to the curve at A, simplifying your answer, [3]

(ii) the equation of the curve. [4]

It is now given that $B\,(16, 8)$ also lies on the curve.

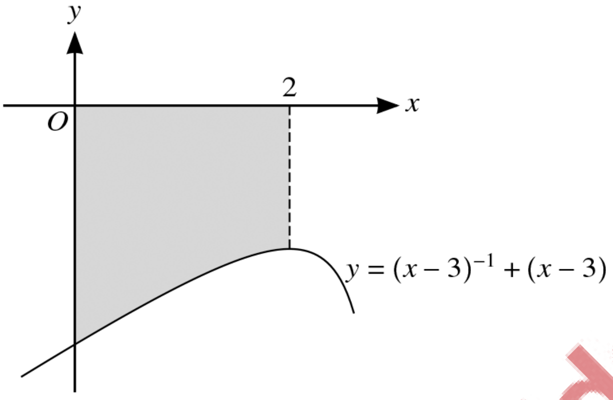
(iii) Find the value of a and, using this value, find the distance AB. [5]

518. 9709_w16_qp_13 Q: 11

A curve has equation $y = (kx - 3)^{-1} + (kx - 3)$, where k is a non-zero constant.

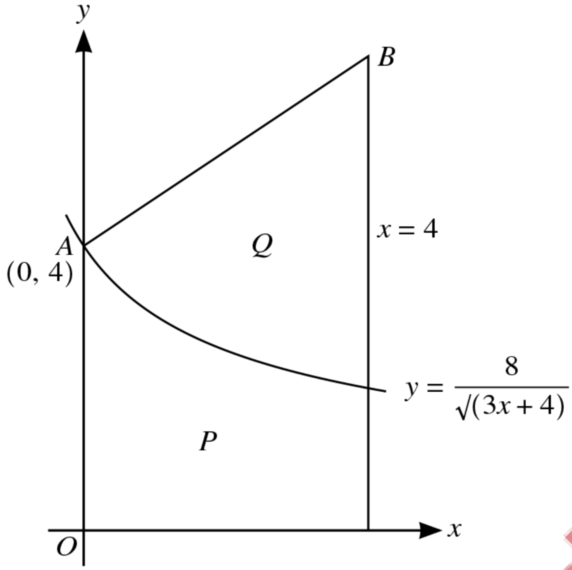
(i) Find the x-coordinates of the stationary points in terms of k, and determine the nature of each stationary point, justifying your answers. [7]

(ii)



The diagram shows part of the curve for the case when $k = 1$. Showing all necessary working, find the volume obtained when the region between the curve, the x-axis, the y-axis and the line $x = 2$, shown shaded in the diagram, is rotated through 360° about the x-axis. [5]

519. 9709_s15_qp_11 Q: 10



The diagram shows part of the curve $y = \dfrac{8}{\sqrt{(3x+4)}}$. The curve intersects the y-axis at $A\,(0, 4)$. The normal to the curve at A intersects the line $x = 4$ at the point B.

(i) Find the coordinates of B. [5]

(ii) Show, with all necessary working, that the areas of the regions marked P and Q are equal. [6]

520. 9709_s15_qp_12 Q: 10

The equation of a curve is $y = \dfrac{4}{2x - 1}$.

(i) Find, showing all necessary working, the volume obtained when the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. [4]

(ii) Given that the line $2y = x + c$ is a normal to the curve, find the possible values of the constant c. [6]

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521. 9709_s15_qp_13 Q: 2

A curve is such that $\dfrac{dy}{dx} = (2x + 1)^{\frac{1}{2}}$ and the point (4, 7) lies on the curve. Find the equation of the curve. [4]

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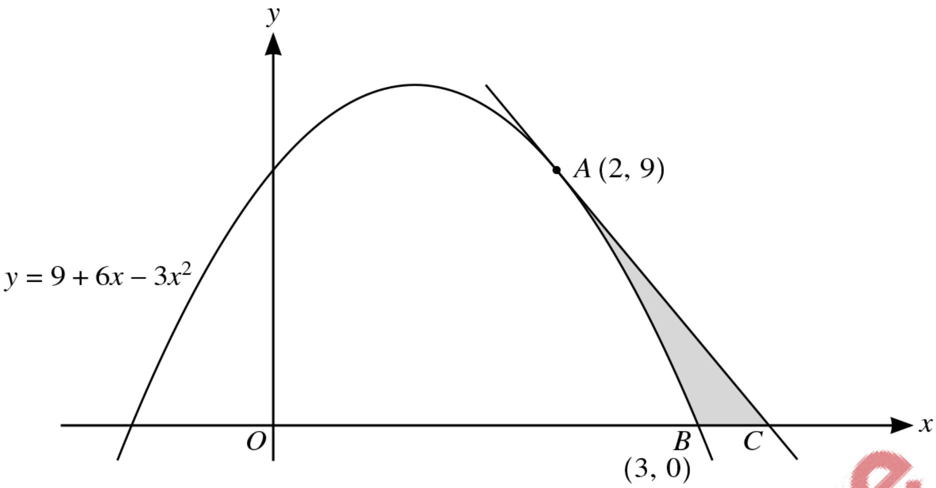
522. 9709_s15_qp_13 Q: 10



Points $A\,(2,\ 9)$ and $B\,(3,\ 0)$ lie on the curve $y = 9 + 6x - 3x^2$, as shown in the diagram. The tangent at A intersects the x-axis at C. Showing all necessary working,

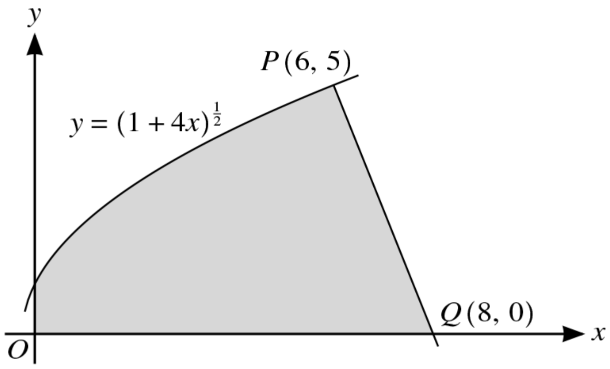
(i) find the equation of the tangent AC and hence find the x-coordinate of C, [4]

(ii) find the area of the shaded region ABC. [5]

523. 9709_w15_qp_11 Q: 2

The function f is such that $f'(x) = 3x^2 - 7$ and $f(3) = 5$. Find $f(x)$. [3]

524. 9709_w15_qp_11 Q: 11



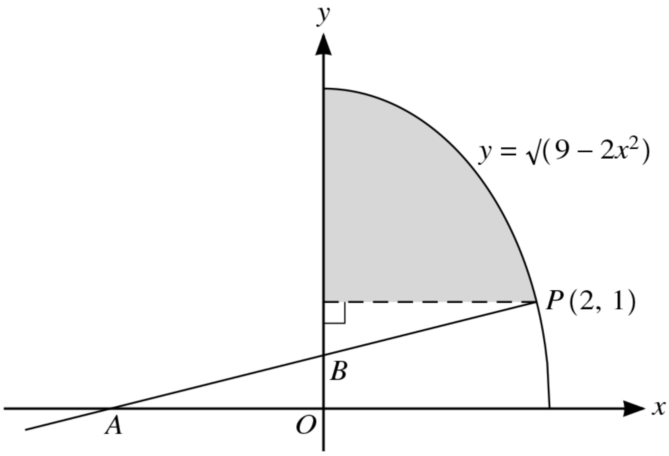
The diagram shows part of the curve $y = (1 + 4x)^{\frac{1}{2}}$ and a point $P\,(6,\,5)$ lying on the curve. The line PQ intersects the x-axis at $Q\,(8,\,0)$.

(i) Show that PQ is a normal to the curve. [5]

(ii) Find, showing all necessary working, the exact volume of revolution obtained when the shaded region is rotated through 360° about the x-axis. [7]

[In part **(ii)** you may find it useful to apply the fact that the volume, V, of a cone of base radius r and vertical height h, is given by $V = \frac{1}{3}\pi r^2 h$.]

525. 9709_w15_qp_12 Q: 10



The diagram shows part of the curve $y = \sqrt{(9 - 2x^2)}$. The point $P(2, 1)$ lies on the curve and the normal to the curve at P intersects the x-axis at A and the y-axis at B.

(i) Show that B is the mid-point of AP. [6]

The shaded region is bounded by the curve, the y-axis and the line $y = 1$.

(ii) Find, showing all necessary working, the exact volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [5]

526. 9709_w15_qp_13 Q: 9

A curve passes through the point $A(4, 6)$ and is such that $\frac{dy}{dx} = 1 + 2x^{-\frac{1}{2}}$. A point P is moving along the curve in such a way that the x-coordinate of P is increasing at a constant rate of 3 units per minute.

(i) Find the rate at which the y-coordinate of P is increasing when P is at A. [3]

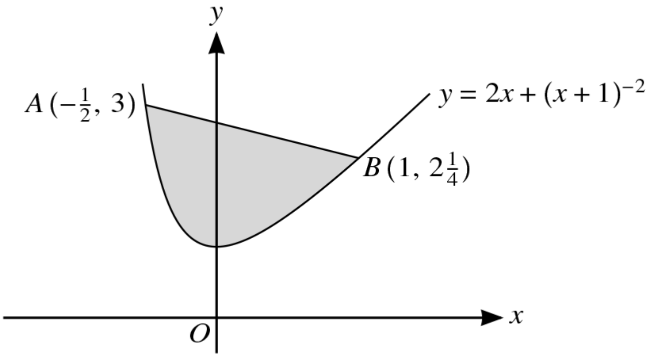
(ii) Find the equation of the curve. [3]

(iii) The tangent to the curve at A crosses the x-axis at B and the normal to the curve at A crosses the x-axis at C. Find the area of triangle ABC. [5]

527. 9709_w15_qp_13 Q: 10

The function f is defined by $f(x) = 2x + (x+1)^{-2}$ for $x > -1$.

(i) Find $f'(x)$ and $f''(x)$ and hence verify that the function f has a minimum value at $x = 0$. [4]



The points $A\,(-\frac{1}{2}, 3)$ and $B\,(1, 2\frac{1}{4})$ lie on the curve $y = 2x + (x+1)^{-2}$, as shown in the diagram.

(ii) Find the distance AB. [2]

(iii) Find, showing all necessary working, the area of the shaded region. [6]